MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 21

Total Marks : 31

Max. Time : 38 min.

Topics : Fundamentals of Mathematics, Function, Limits

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1	**(3 marks, 3 min.)**	**[3, 3]**
Subjective Questions (no negative marking) Q.2,3,4,5,6,7,8	**(4 marks, 5 min.)**	**[28, 35]**

1. Given $x^2 - xy + y^2 = 4\,(x + y - 4)$, where x, y both are real numbers. The number of pairs (x, y) satisfying the equation is

(A) only one (B) only two (C) three (D) None of these

2. Evaluate

(i) $\lim\limits_{n\to\infty} (2^n + 3^n)^{1/n}$ (ii) $\lim\limits_{x\to -1} \dfrac{\cos 2 - \cos 2x}{x^2 - |x|}$

3. Evaluate

(i) $\lim\limits_{x\to 0} \dfrac{\tan\sqrt[3]{x}\ \ell n(1+3x)}{(\tan^{-1}\sqrt{x})^2(e^{5\sqrt[3]{x}} - 1)}$ (ii) $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{x} + \sqrt{x} + x\sqrt{x} - 3}{x^3 - 1}$

4. (a) $\lim\limits_{x\to\infty} \left(\dfrac{x-2}{x^2 - 4x + 3}\right)^x$ is equal to

(b) $\lim\limits_{x\to 2} [x]$ (where [.] denotes greatest integer function) is equal to

(c) $\left[\lim\limits_{x\to 2} x\right]$ (where [.] denotes greatest integer function) is equal to

5. Solve $\dfrac{1}{[x]} + \dfrac{1}{[2x]} = \{x\} + \dfrac{1}{3}$, where [.] denotes greatest integral function and {x} denotes fractional part of x.

6. (a) Whether function $f(x) = \begin{cases} x & 0 \le x < 1 \\ 3 - x & 1 \le x \le 2 \end{cases}$ is invertible ? If yes, then find its inverse.

(b) If Domain of f(x) is $[\pi, 3\pi)$ & $g(x) = \pi + x + \sin x$, then find domain of f(g(x)).

7. Evaluate :

(i) $\lim\limits_{x\to\infty} \dfrac{(2+x)^{40}\ (4+x)^5}{(2-x)^{45}}$ (ii) $\text{Lim}\limits_{x\to 0} \dfrac{1 - \cos^3 x}{x \sin x \cos x}$ (iii) $\text{Lim}\limits_{x\to 0} \dfrac{\ell n(1+2x) - 2\ell n(1+x)}{x^2}$

8. Evaluate :

(i) $\lim\limits_{x\to 0} \dfrac{x^2 2^{2x} - x^2.2^{x+1} + x^2}{\cos 2x - 4\cos x + 3}$ (ii) $\lim\limits_{x\to\infty} \dfrac{(x+1)^4 - (x-1)^4}{(x+1)^4 + (x-1)^4}$

Answers Key

1. (A) **2.** (i) 3 (ii) 2 sin 2 **3.** (i) $\frac{3}{5}$ (ii) $\frac{7}{9}$

4. (a) 0 (b) does not exist (c) 2

5. $\frac{29}{12}, \frac{19}{6}, \frac{97}{24}$

6. (a) Yes, $f^{-1}(x) = \begin{cases} x, & 0 \le x < 1 \\ 3 - x, & 1 \le x \le 2 \end{cases}$ (b) $[0, 2\pi)$

7. (i) – 1 (ii) $\frac{3}{2}$ (iii) – 1

8. (i) $2(\ell n\, 2)^2$ (ii) 0